1 a When $t = 0, x = 12$.
12 cm to the right of O

b $\text{When } t = 5, x = 5^2 - 7 \times 5 + 12$
$= 2$
2 cm to the right of O

c $v = \frac{dx}{dt}$
$= 2t - 7$
When $t = 0, v = -7$.

7cm/s to the left

d $v = 0 \text{ when } 2t - 7 = 0$
$t = 3.5$
When $t = 3.5$,
$x = 3.5^2 - 7 \times 3.5 + 12$
$= -0.25$
$t = 3.5$; the particle is 0.25 cm to the left of O.

e $\text{Average velocity} = \frac{\text{change in position}}{\text{change in time}}$
$= \frac{2 - 12}{5}$
$= -2 \text{ cm/s}$

f $\text{Average speed} = \frac{\text{distance travelled}}{\text{change in time}}$
For the first 3.5 s, the particle has travelled 12.25 cm.
From 3.5 s to 5 s, the particle has travelled $2 - (-0.25) = 2.25$ cm.
$\text{Average speed} = \frac{12.25 + 2.25}{5}$
$= \frac{14.5}{5}$
$= 2.9 \text{ cm/s}$

2 a $v = \frac{dx}{dt}$
$= 2t - 7$
$v = 0 \text{ when } 2t - 7 = 0$
$t = 3.5 \text{ s}$

b $a = \frac{dv}{dt}$
$= 2 \text{ m/s}^2$

c When $t = 0, x = 10$.
$\text{When } t = 3.5, x = 3.5^2 - 7 \times 3.5 + 10$
$= -2.25$
For the first 3.5 s, the particle has travelled 12.25 m.
$\text{When } t = 5, x = 5^2 - 7 \times 5 + 10$
$= 0$
From 3.5 s to 5 s, the particle has travelled 2.25 m.

$$\text{Distance travelled} = 12.25 + 2.25$$
$$= 14.5 \text{ m}$$

d $v = 2t - 7 = -2$

$2t = 5$

$t = 2.5$

$x = 2.5^2 - 7 \times 2.5 + 19$

$= -1.25$

After 2.5 s, when the particle is 1.25 m left of O.

3 a When $t = 0, x = -3$.

$$v = \frac{dx}{dt}$$
$$= 3t^2 - 22t + 24$$

When $t = 0, v = 24$.

3 cm to the left of O and moving at 24 cm/s to the right.

b $$v = \frac{dx}{dt}$$
$$= 3t^2 - 22t + 24$$

c $v = 0$ when

$3t^2 - 22t + 24 = 0$

$(3t - 4)(t - 6) = 0$

$$t = \frac{4}{3} \text{ or } 6$$

After $\frac{4}{3}$ s and after 6 s

d When $t = \frac{4}{3}$,

$$x = \left(\frac{4}{3}\right)^3 - 11 \times \left(\frac{4}{3}\right)^2 + 24 \times \left(\frac{4}{3}\right) - 3$$
$$= \frac{64}{27} - \frac{176}{9} \times \frac{3}{3} + 32 - 3$$
$$= -\frac{464}{27} + 29$$
$$= 11\frac{22}{27}$$

When $t = 6$,

$$x = 6^3 - 11 \times 6^2 \times 6 - 3$$
$$= -39$$

39 cm to the left of O and 11 $\frac{22}{27}$ cm to the right of O

e $v < 0$ when $(3t - 4)(t - 6) = 0$

This is a parabola with a minimum value.

$$\therefore \quad v < 0 \text{ when } \frac{4}{3} < t < 6$$

$$\text{Length of time} = 6 - \frac{4}{3}$$
$$= \frac{14}{3}$$
$$= 4\frac{2}{3} \text{ s}$$

f $a = \dfrac{dv}{dt}$

$= 6t - 22 \text{ m/s}^2$

g $6t - 22 = 0$

$$t = \frac{22}{6} = \frac{11}{3}$$

$$v = 3t^2 - 22t + 24$$

$$= 3 \times \left(\frac{11}{3}\right)^2 - 22 \times \frac{11}{3} + 24$$

$$= \frac{121}{3} - \frac{242}{3} + 24$$

$$= 16\,\frac{2}{3}$$

$$x = \left(\frac{11}{3}\right)^3$$

$$= 11 \times \left(\frac{11}{3}\right)^2 + 24 \times \frac{11}{3} - 3$$

$$= \frac{1331}{27} - \frac{1331}{9} \times \frac{3}{3} + 88 - 3$$

$$= -13\,\frac{16}{27}$$

The acceleration is zero after $\frac{11}{3}$ s, when the velocity is 16 $\frac{1}{3}$ cm/s to the left and its position is 13 $\frac{16}{27}$ cm left of O.

4 **a** $v = 6t^2 - 10t + 4$

When $v = 0$:

$$6t^2 - 10t + 4 = 0$$

$$3t^2 - 5t + 2 = 0$$

$$(3t - 2)(t - 1) = 0$$

$$t = \frac{2}{3} \text{ or } 1$$

$a = 12t - 10$

$t = \frac{2}{3}$:

$a = 12 \times \frac{2}{3} - 10$

$= -2$

$t = 1$:

$a = 12 \times 1 - 10$

$= 2$

Velocity is zero after $\frac{2}{3}$ s when the acceleration is 2 cm/s^2 to the left, and after 1 s when the acceleration is 2 cm/s^2 to the right.

b $a = 12t - 10$

$= 0$

$t = \frac{10}{12} = \frac{5}{6}$

Find v when $a = \frac{5}{6}$:

$v = 6t^2 - 10t + 4$

$$= 6 \times \left(\frac{5}{6}\right)^2 - 10 \times \frac{5}{5} + 4$$
$$= \frac{25}{6} - \frac{50}{6} + 4 = -\frac{1}{6}$$

Acceleration is zero after $\frac{5}{6}$ s, at which time the velocity is $\frac{1}{6}$ cm/s to the left.

5 The particle passes through O when $x = 0$.

$$t^3 - 13t^2 + 46t - 48 = 0$$

Trial and error will give $x = 0$ when $t = 2$.

This means $(t-2)$ is a factor of $t^3 - 13t^2 + 46t - 48$.

$$\begin{aligned} &t^3 - 13t^2 + 46t - 48 \\ &= (t-2)(t^2 - 11t + 24) \\ &= 0 \end{aligned}$$

Factorising the quadratic gives

$$(t-2)(t-3)(t-8) = 0$$

$$\begin{aligned} t &= 2, 3 \text{ or } 8 \\ v &= \frac{dx}{dt} \\ &= 3t^2 - 26t + 46 \\ a &= \frac{dv}{dt} \\ &= 6t - 26 \end{aligned}$$

$t = 2:$

$$\begin{aligned} v &= 3 \times 4 - 26 \times 2 + 46 \\ &= 6 \text{ cm/s} \\ a &= 6 \times 2 - 26 \\ &= -14 \text{ cm}^2\text{/s} \end{aligned}$$

$t = 3:$

$$\begin{aligned} v &= 3 \times 9 - 26 \times 3 + 46 \\ &= -5 \text{ cm/s} \\ a &= 6 \times 3 - 26 \\ &= -8 \text{ cm}^2\text{/s} \end{aligned}$$

$t = 8:$

$$\begin{aligned} v &= 3 \times 64 - 26 \times 8 + 46 \\ &= 30 \text{ cm/s} \\ a &= 6 \times 8 - 26 \\ &= -22 \text{ cm}^2\text{/s} \end{aligned}$$

6 a They will be at the same position when

$$\begin{aligned} t^2 - 2t - 2 &= t + 2 \\ t^2 - 3t - 4 &= 0 \\ (t-4)(t+1) &= 0 \\ t &= 4 \text{ or } -1 \end{aligned}$$

After 4 s, or 1 s before the start.

(Note: In some cases, motion is not considered before $t = 0$, and negative values of t may be discarded.)

b The velocities are $1\ \mathrm{cm/s}$ and $2t-2\ \mathrm{cm/s}$.

$$\begin{aligned} 2t-2 &= 1 \\ 2t &= 3 \\ t &= \frac{3}{2} \end{aligned}$$

After $\frac{3}{2}$ s.